INFORMATION FLOW IN THE RETINA-CORTEX PATH:
GAUSSIANIZATION ESTIMATES & THEORETICAL RESULTS

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UNIVERSITAT DE VALÈNCIA
SPAIN

CNS* 2020 Workshop on Methods of Information Theory in Computational Neuroscience
1. **IMAGE QUALITY & VISION MODELS**: Computational Neuroscience may make you a MOVIE STAR!

2. **THE PROBLEM**: Quantifying information flow

3. **THE PROPOSED TECHNIQUE**: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

4. **EXPERIMENTS & RESULTS**: Efficient Coding & Image Quality

5. **DISCUSSION & CONCLUSIONS**
IMAGE QUALITY & VISION MODELS: Computational Neuroscience may make you a MOVIE STAR!
Structural Similarity (SSIM)

67th EMMY Engineering Award of the American Television Academy 2015!

https://youtu.be/e5-LCFGdgMA
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Image Quality & Vision Models: Computational Neuroscience may make you a MOVIE STAR!

Google Scholar

Alan Bovik
Cockrell Family Regents Endowed Chair Professor, The University of Texas at Austin
Verified email at ece.utexas.edu - Homepage
Electrical Engineering Digital Television Digital Photography Social Media Image Processing

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<tr>
<th>TITLE</th>
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<td>Image quality assessment: from error visibility to structural similarity</td>
<td>26470</td>
<td>2004</td>
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<td>Z. Wang, AC. Bovik, HR. Sheikh, EP. Simoncelli</td>
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<td>IEEE Transactions on Image Processing 13 (4), 606-612</td>
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<td>A universal image quality index</td>
<td>5361</td>
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<td>Z. Wang, AC. Bovik</td>
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<td>IEEE Signal Processing Letters 9 (3), 81-84</td>
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<td>Image information and visual quality</td>
<td>2959</td>
<td>2006</td>
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<td>HR. Sheikh, AC. Bovik</td>
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<td>IEEE Transactions on Image Processing 15 (2), 430-444</td>
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Cited by

Citations 100508 53293
h-index 114 77
h10-index 478 304

Co-authors

Zhou Wang
Professor, Electrical and Compu...
Image Quality Assessment: From Error Visibility to Structural Similarity

Zhou Wang, Member, IEEE, Alan C. Bovik, Fellow, IEEE
Hamid R. Sheikh, Student Member, IEEE, and Eero P. Simoncelli, Senior Member, IEEE

Image Information and Visual Quality

Hamid Rahim Sheikh, Member, IEEE, and Alan C. Bovik, Fellow, IEEE

\[ d(C, D) = |S(C) - S(D)| = |E - F| \]

GEOMETRY \equiv DISTANCE
Image Quality Assessment: From Error Visibility to Structural Similarity

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A. New Philosophy

\[ \text{SSIM} \propto \frac{s_{xy} + k}{s_x s_y + k} \]

Fig. 3. Diagram of the structural similarity (SSIM) measurement system.

\[ \text{GEOMETRY} \equiv \text{DISTANCE} \]

\[ d(c, d) = |s(c) - s(d)| = |e - f| \]
Image Quality Assessment: From Error Visibility to Structural Similarity

Zhou Wang, Member, IEEE, Alan C. Bovik, Fellow, IEEE
Hamid R. Sheikh, Student Member, IEEE, and Eero P. Simoncelli, Senior Member, IEEE

A. New Philosophy

**CLASSIFICATION**

\[ \text{SIM} \propto \frac{\sigma_{xy} + k}{\sigma_x \sigma_y + k} \]

Fig. 3. Diagram of the structural similarity (SSIM) measurement system.

Image Information and Visual Quality

Hamid Rahim Sheikh, Member, IEEE, and Alan C. Bovik, Fellow, IEEE

A. New Philosophy

\[ \text{GEOGRAPHY} \equiv \text{DISTANCE} \]

\[ d(C, D) = |S(C) - S(D)| = |E - F| \]

\[ \text{VIF} = \frac{I(C, S(D))}{I(C, S(C))} = \frac{I(C, E)}{I(C, E)} \]
**IMAGE QUALITY & VISION MODELS:** Computational Neuroscience may make you a MOVIE STAR!

- **RMSE** (Euclidean Distance)
- **SSIM**

**SSIM**
67th EMMY Engineering Award of the American Television Academy 2015
1. IMAGE QUALITY & VISION MODELS: Computational Neuroscience may make you a MOVIE STAR!

RMSE
Euclidean Distance

V1_model_DCT_DN_color
Im. Vis. Comp. 1997
IEEE Trans. Im. Proc. 2006...

V1_model_wavelet_DN_color
JOSA A 2010
Neural Comput. 2010

BioMultiLayer_L_NL_color
Front. Neurosci. 2018 a

BioMultiLayer_L_NL_color (Optimized)
PLoS ONE 2018

 SSIM
67th EMMY Engineering Award of the American Television Academy 2015

VIF

Divisive Normalization
Teo & Heeger 94
Malo et al. 97
Malo & Simoncelli 06
Laparra & Malol 10
Malo & Simoncelli 15
Laparra & Simoncelli 17
Malo et al. 18
Hepburn & Malol 20
Divisive Normalization neural models

\[ S \]

\[ x \rightarrow \mathcal{L} \rightarrow e \rightarrow N \rightarrow y \]

BION: Carandini & Heeger Nat. Rev. Neurosci: 12

MATH: Martinez, Malo et al. PLOS ONE 18
Divisive Normalization neural models

\[ S \]
\[ x \rightarrow e \rightarrow N \rightarrow y \]

\[ e = W \cdot x \]

BIOI: Carandini & Heeger, Nat. Rev. Neurosci: 12
MATH: Martinez, Malo et al., PLOS ONE 18
**Image Quality & Vision Models**: Computational Neuroscience may make you a MOVIE STAR!

- **Divisive Normalization Neural Models**
- **BIOL. Carandini & Heeger** Nat. Rev. Neurosci. 12
- **MATH. Martinez, Malo et al.** PLOS ONE 18

\[ S \]
\[ x \rightarrow z \rightarrow e \rightarrow N \rightarrow y \]

\[ e = W \cdot x \]

\[ y = k \cdot \frac{e}{b + H \cdot e} \]

\[ \nabla_x S \sim (I - D_{e}(H)) \cdot D_e \cdot W \Rightarrow M = \nabla_x S^T \nabla_x S \]

\[ e_j = 0 \quad e_j \gg \]

Masking and adaptation

- **Masking and adaptation**
- Non diagonal!
- Input dependent!
IMAGE QUALITY & VISION MODELS: Computational Neuroscience may make you a MOVIE STAR!

BIOL. Carandini & Heeger Nat. Rev. Neurosci: 12
MATH. Martinez, Malo et al. PLOS ONE 18
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2) **THE PROBLEM:** Quantifying information flow
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2) THE PROBLEM: Quantifying information flow

ACADEMY OF TELEVISION ARTS & SCIENCES

DARWIN

BARLOW
THE PROBLEM: Quantifying information flow
2) THE PROBLEM: Quantifying information flow

\[ x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \]

\[ S_\theta \]

\[ y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \]

\[ h(x_1) \]

\[ h(x_2) \]

\[ h(y_1) \]

\[ h(y_2) \]
2) THE PROBLEM: Quantifying information flow

\[ x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \xrightarrow{S_0} \quad y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \]

\[ h(x_1) \quad h(x_2) \quad h(y_1) \quad h(y_2) \]

\\

\[ \text{TOTAL CORRELATION = Redundancy within a vector} \quad T(y) = \sum_{i} h(x_i) - h(y) \]
(2) THE PROBLEM: Quantifying information flow

\[ x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \xrightarrow{S_0} \quad y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \]

\[ h(x_1) \quad h(x_2) \quad h(y_1) \quad h(y_2) \]

\[ T(y) = \sum_i h(y_i) - h(y) \]

/// TOTAL CORRELATION = Redundancy within a vector \[ T(y) = \sum_i h(y_i) - h(y) \]
2) THE PROBLEM: Quantifying information flow

\[ x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad S_6 \quad y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \]

\[ h(x_i) \quad h(y_i) \quad h(x_2) \quad h(y_2) \]

\[ T(y) = \sum_i h(y_i) - h(y) \]

\[ I(x,y) = h(x) + h(y) - h(x,y) \]
THE PROBLEM: Quantifying information flow

THE EFFICIENT CODING HYPOTHESIS
(information maximization)
Barlow 59, Barlow 01

RETINA

V1 CORTEX
(2) THE PROBLEM: Quantifying information flow

THE EFFICIENT CODING HYPOTHESIS

(information maximization)

Barlow 59, Barlow 01

\[ y = S(x) + n \]

\[ I(x, y) = \sum_{i} h(y_i) - \bar{T}(y) - h(n) \Rightarrow \]

\{ (1) maximize entropy, (2) minimize redundancy \}

THE PROBLEM: Quantifying information flow

THE EFFICIENT CODING HYPOTHESIS (information maximization)
Barlow 59, Barlow 01

\[ y = S(x) + \eta \]

\[ I(x; y) = \sum_i h(y_i) - H(y) - h(\eta) \Rightarrow \begin{cases} 
(1) \text{ maximize entropy} \\
(2) \text{ minimize redundancy} 
\end{cases} \]

THE PROBLEM: Quantifying information flow

THE EFFICIENT CODING HYPOTHESIS

- Standard
- Alternative

STATS → BIOLOGY

BIOLOGY → STATS

PROBLEM: Estimating $I(x, y)$ from samples = THE CURSE OF DIMENSIONALITY
The proposed technique: Rotation-Based Iterative Gaussianization (RBIG)
THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

Laparra, Camps, Malo IEEE TNN 2011
Johnson, Laparra, Malo ICML 2019

https://isp.uv.es/RBIG4IT.htm
The proposed technique: Rotation-Based Iterative Gaussianization (RBIG)

Any PDF: \( p(x^{(0)}) \)

Gaussian PDF: \( p(x^{(N)}) = N(x^{(N)}, 0, I) \)
The proposed technique: Rotation-Based Iterative Gaussianization (RBIG)

\[ x^{(u+1)} = R^{(u)} \cdot \Psi^{(u)}(x^{(u)}) \]

Rotation
Marginal Gaussianization
The proposed technique: Rotation-Based Iterative Gaussianization (RBIG)

\[ x^{(0)} \rightarrow x^{(1)} \rightarrow x^{(2)} \]

ANY PDF

\[ P(x^{(0)}) \]

\[ x^{(u+1)} = R^{(u)} \cdot \Psi^{(u)}(x^{(u)}) \]

- Rotation
- Marginal Gaussianization
ANY PDF $p(x^{(0)})$

$x^{(0)} \rightarrow x^{(1)} \rightarrow x^{(2)} \rightarrow x^{(3)}$

$x^{(n+1)} = R^{(n)} \cdot \psi^{(n)}(x^{(n)})$

- Rotation
- Marginal Gaussianization
The proposed technique: Rotation-Based Iterative Gaussianization (RBIG)

\[ x^{(0)} \rightarrow x^{(1)} \rightarrow x^{(2)} \rightarrow x^{(3)} \rightarrow \ldots \rightarrow x^{(N)} \]

ANY PDF

\[ p(x^{(0)}) \]

\[ x^{(n+1)} = R^{(n)} \cdot \Psi^{(n)}(x^{(n)}) \]

GAUSSIAN PDF

\[ p(x^{(N)}) = N(x^{(N)}, 0, I) \]

Rotation

Marginal Gaussianization
The proposed technique: Rotation-Based Iterative Gaussianization (RBIG)

\[ x^{(u+1)} = R^{(u)} \cdot \Psi^{(u)}(x^{(u)}) \]

Rotation  Marginal Gaussianization
In ANY differentiable transform

\[ \Delta T(x, x') = T(x) - T(x') \]
\[ \Delta T(x, x') = \sum_{i=1}^{D_x} H(x_i) - \sum_{i=1}^{D_{x'}} H(x_i') + \frac{1}{2} \mathbb{E}_x \left( \log |\nabla G_x(x)^T \cdot \nabla G_x(x)| \right) \]

In ANY Gaussianization \( T(x') = 0 \)

\[ T(x) = \sum_{i=1}^{D_x} H(x_i) - \frac{D_x}{2} \log(2\pi e) + \mathbb{E}_x \left( \log |\nabla G_x(x)| \right) \]
THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

In ANY differentiable transform \( \Delta T(x, x') = T(x) - T(x') \)
\[
= \sum_{i=1}^{D_x} H(x_i) - \sum_{i=1}^{D_x'} H(x'_i) + \frac{1}{2} \mathbb{E}_x \left( \log \left| \nabla G_x(x)^T \cdot \nabla G_x(x) \right| \right)
\]

\( T(x) = \sum_{i=1}^{D_x} H(x_i) - \frac{D_x}{2} \log(2\pi e) + \mathbb{E}_x \left( \log \left| \nabla G_x(x) \right| \right) \)

IN RBIG = ONLY UNIVARIATE OPERATIONS

\[
\tilde{T}(x) = \sum_{n=0}^{N-1} \Delta \tilde{T}^{(n)} = \frac{(N-1)D_x}{2} \log(2\pi e) - \sum_{n=1}^{N} \sum_{i=1}^{D_x} \tilde{H}(x_i^{(n)})
\]
THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

\[ \Delta T(x, x') = T(x) - T(x') \]
\[ = \sum_{i=1}^{D_x} H(x_i) - \sum_{i=1}^{D_{x'}} H(x'_i) + \frac{1}{2} E_x (\log |\nabla G_x(x)^T \cdot \nabla G_x(x)|) \]
\[ T(x) = \sum_{i=1}^{D_x} H(x_i) - \frac{D_x}{2} \log(2\pi e) + E_x (\log |\nabla G_x(x)|) \]

\[ \tilde{T}(x) = \sum_{n=0}^{N-1} \Delta \tilde{T}^{(n)} = \frac{(N-1)D_x}{2} \log(2\pi e) - \sum_{n=1}^{N} \sum_{i=1}^{D_x} \tilde{H}(x_i^{(n)}) \]

In ANY differentiable transform \( \Rightarrow \) In ANY Gaussianization \( T(x') = 0 \)

\[ \quad \flipblue {\text{CURSE OF DIMENSION, ALLEVIATED!}} \]
Total Correlation

\[ \hat{T}(x) = \sum_{n=0}^{N-1} \Delta \tilde{T}^{(n)} = \frac{(N-1)D_x}{2} \log(2\pi e) - \sum_{n=1}^{N} \sum_{i=1}^{D_x} \hat{H}(x_i^{(n)}). \]

Differential Entropy

\[ \hat{H}(x) = \sum_{i=1}^{D_x} \hat{H}(x_i) - \tilde{T}(x) \]

Kullback-Leibler Div.

\[ D_{KL}(y|x) = D_{KL}(G_x(y)|G_x(x)) = T(x) + \sum_{i=1}^{D_x} D_{KL}(p_{x_i}(x_i)|\mathcal{N}(0,1)) \]

Mutual Information

\[ \tilde{I}(x, y) = \hat{T}([G_x(x), G_y(y)]) \]
3. THE PROPOSED TECHNIQUE: ROTATION-BASED ITERATIVE GAUSSIANIZATION (RBIG)

Total Correlation

\[ \tilde{T}(x) \]

\[ \tilde{H}(x) \]

\[ D_{KL}(y|x) \]

\[ \tilde{I}(x,y) \]

Kullback-Leibler Div.

Differential Entropy

Szabo JMLR 2014
KNN
Partition Trees
Exp. Family
Von Mises
Ensemble

https://isp.uv.es/RBIG4IT.htm

Mutual Information
3) **The Proposed Technique:** Rotation-Based Iterative Gaussianization (RBIG)

### Total Correlation

<table>
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<tr>
<th>D_x</th>
<th>RBIG</th>
<th>LNN</th>
<th>KDP</th>
<th>expf</th>
<th>rMI</th>
<th>Emn</th>
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<td>0.97</td>
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<td>63.85</td>
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<td>100</td>
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<td>72.36</td>
<td>66.99</td>
<td>59.94</td>
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\[ \hat{D}(x) \]

### Differential Entropy

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<th>rMI</th>
<th>Emn</th>
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<td>10.07</td>
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\[ \hat{H}(x) \]

### Kullback-Leibler Div.

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\[ D_{KL}(y|x) \]

### Mutual Information

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\[ \hat{I}(x,y) \]

### RBIG Info-Theory Measures Work!

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**Szabo JMLR 2014**
**KNN**
**Partitions**
**Exp. Family**
**Von Mises**
**Ensemble**

[https://isp.uv.es/RBIG4IT.htm](https://isp.uv.es/RBIG4IT.htm)
4. EXPERIMENTS & RESULTS
EXPERIMENTS & RESULTS

Materials
- Natural Images
- Vision model: Retina - Cortex
- Performance
- Assumptions

Experiments Efficient Coding
- Global - $\Delta T(x,y)$  
- $I(x,y)$
- Local - $\Delta T(x,y)$  
- $I(x,y)$

Layers & flexibility

PDF

Experiments Image Quality
4. EXPERIMENTS & RESULTS  Material: Natural images

Laparra & Malo *Neural Comp.* 2012, Gutmann & Malo *PLOS* 2014 [https://isp.uv.es/data_color.htm](https://isp.uv.es/data_color.htm)

Probability Density Function of Natural Images

EXPERIMENTS & RESULTS  Material: Natural Images
### Experiments & Results

#### Material: Vision Model

<table>
<thead>
<tr>
<th>Spatial Extent</th>
<th>$r^{(1)}$</th>
<th>$r^{(2)}$</th>
<th>$x^{(2)}$</th>
<th>$r^{(3)}$</th>
<th>$x^{(3)}$</th>
</tr>
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<tbody>
<tr>
<td>More flexible model</td>
<td>(0.27 deg)</td>
<td>0.38</td>
<td>0.38</td>
<td>0.42</td>
<td>0.77</td>
</tr>
<tr>
<td>Baseline model</td>
<td>(0.27 deg)</td>
<td><strong>0.38</strong></td>
<td>0.38</td>
<td>0.42</td>
<td>0.77</td>
</tr>
<tr>
<td>More rigid model</td>
<td>(0.27 deg)</td>
<td>0.38</td>
<td>0.38</td>
<td>0.42</td>
<td>0.77</td>
</tr>
<tr>
<td>Totally rigid model</td>
<td>(0.27 deg)</td>
<td>0.38</td>
<td>0.38</td>
<td>0.38</td>
<td>0.68</td>
</tr>
<tr>
<td>Baseline model</td>
<td>(0.05 deg)</td>
<td>0.26</td>
<td>0.27</td>
<td>0.31</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Pearson correlation with human viewers using different building blocks (or model layers)

![Graph with data points and correlation coefficients](image-url)
4. EXPERIMENTS & RESULTS

Material: Vision Model

```
Distance at Linear LMS
ρ_p = 0.38  ρ_s = 0.49  ρ_k = 0.35

Distance at Linear ATD
ρ_p = 0.38  ρ_s = 0.48  ρ_k = 0.34

Distance at Nonlinear ATD
ρ_p = 0.42  ρ_s = 0.52  ρ_k = 0.36

Distance at Linear DCT-CSF
ρ_p = 0.77  ρ_s = 0.79  ρ_k = 0.61

Distance at Nonlinear Div. Norm.
ρ_p = 0.84  ρ_s = 0.85  ρ_k = 0.67
```

Pearson correlation with human viewers using different building blocks (or model layers)

<table>
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<tr>
<th>Spatial Extent</th>
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<td>0.05</td>
<td>0.26</td>
<td>0.27</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Assumptions:

1. Single-step transforms
2. Constant SNR (5% noise)
Efficient Coding (global)

Redundancy Reduction
\( \Delta T(x_{inp}^{+}, x_{resp}) \)

Pearson correlation with human viewers using different building blocks (or model layers)

<table>
<thead>
<tr>
<th>Spatial Extent</th>
<th>( r^{(1)} )</th>
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Transmitted Information
\( I(x_{inp}, x_{resp}) \)
4. EXPERIMENTS & RESULTS

Redundancy Reduction

\[ \Delta T(x^{inp}, x_{\text{resp}}) \]

\[ \Delta T_{\text{BBG}} = \sum_i h(x_i^{inp}) - h(x_i^{resp}) + E_x \left[ \log_2 |\nabla s| \right] \]

Pearson correlation with human viewers using different building blocks (or model layers)

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Transmitted Information

\[ I(x^{inp}, x_{\text{resp}}) \]
4. EXPERIMENTS & RESULTS

Redundancy Reduction

\[ \Delta T(x_{\text{in}}, x_{\text{resp}}) \]

\[ \Delta T_{\text{BBIG}} = \sum_i h(x_i^{\text{in}}) - h(x_i^{\text{resp}}) + E_x \left[ \log_2 |\nabla s| \right] \]

Pearson correlation with human viewers using different building blocks (or model layers)

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\[ I(x_{\text{in}}, x_{\text{resp}}) \]
**Experiments & Results**

Redundancy Reduction

\[ \Delta T(x^{in^{t}}, x_{res}) \]

\[ \Delta T_{BBIG} = \sum_{i} h(x^{i^{n}}) - h(x_{res}^{n}) + E_x \left[ log_2 |\Delta SL| \right] \]

Pearson correlation with human viewers using different building blocks (or model layers):

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- Deeper is better
- Baseline is better
- Space vs Color

Transmitted Information

\[ I(x^{in}, x_{res}) \]
Efficient Coding (local)
Redundancy Reduction

RBIG estimates work!
4. EXPERIMENTS & RESULTS

Efficient Coding (local)

Transmitted Information

PDF

Natural Images

Transmitted info matches PDF
4 Experiments & Results

HVS

Natural Image Source

Channel (Distortion)

\[ VIF = \frac{I(C, S(D))}{I(C, S(c))} = \frac{I(C, F)}{I(C, E)} \]
**Experiments & Results**

**Visual Information Fidelity**

\[
\text{Conventional VIF} = \sum \frac{\log_2 \left( \frac{|s_i \cdot C_u + (s_i^2 + s_i^2) \cdot \mathbb{I}|}{|s_i^2| \cdot \mathbb{I}} \right)}{\sum_i \log_2 \left( \frac{|s_i \cdot C_u + s_i^2|}{|s_i^2|} \right)}
\]

* Vision Model: Wavelet + Gauss. Noise
* Mutual Inform. Assumes G.S.M.
**4) EXPERIMENTS & RESULTS**

**Visual Information Fidelity**

Conventional VIF

* Vision Model: Wavelet + Gauss. Noise

* Mutual Inform. Assumes G.S.M.

\[
VIF = \frac{I(C, S(D))}{I(C, S(C))} = \frac{I(C, F)}{I(C, E)}
\]

Alternative VIF:

* Vision Model: Wavelet + Gauss. Noise

* Compute \( I(C, F) \) empirically using RBIG
4 Experiments & Results

Visual Information Fidelity

Conventional VIF

\[
\text{VIF} = \frac{I(C,S(D))}{I(C,S(C))} = \frac{I(C,F)}{I(C,E)} = \frac{\sum_i \log_2 \left( \frac{|s_i C + (\sigma_i^2 + \sigma_{s_i}^2)|}{1 + \sigma_i^2} \right)}{\sum_i \log_2 \left( \frac{|s_i C + \sigma_{s_i}^2|}{|C_s|} \right)}
\]

Alternative VIF:

\* Vision Model: Wavelet + Gauss. Noise
\* Compute \( I(C,F) \) \( I(C,E) \) empirically using RBIG

Correlation with Subjective Opinion (TID 2013)

<table>
<thead>
<tr>
<th></th>
<th>Pearson</th>
<th>Spearman</th>
<th>Kendall</th>
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<tbody>
<tr>
<td>Conventional VIF</td>
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<td>0.76</td>
<td>0.60</td>
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<tr>
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<td>0.81</td>
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<td>0.59</td>
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<tr>
<td>Alternative RBIG-VIF</td>
<td>0.90</td>
<td>0.87</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Compt. by Benjamin Khervodal
DISCUSSION & CONCLUSIONS
**Discussion & Conclusions**

- Information theory estimates & curse of dimensionality
- Gaussianization $\Rightarrow T(x)$, but still requires $E_x \left[ \log_2 D \right]
- RBIG $\Rightarrow T(x)$ $\sim$ univariate operations $\Rightarrow$ Alleviates curse of dim.
- RBIG $\Rightarrow T$, H, $D_{KL}$, I better than Szabo JMLR 2014
- **Efficient Coding Hypothesis in Psychophysical Models**
  - Deeper represent. are better
  - Psychophysically accurate models are better
  - Space more important than color
  - Transmitted information matches PDF of natural images
- **Image Quality**: RBIG empirical estimates of I improve VIF
Better information estimates (e.g. using RBIG)

Image Quality Problem

Efficient Coding Hypothesis
Better information estimates (e.g. using RBIG)

Questions?

Comments?