GEOMETRY OF THE SPACE OF VISUAL STIMULI: PSYCHOPHYSICS AND NEURAL MODELS

WORKSHOP: GEOMETRY OF COLOR PERCEPTION, CORTICAL MODELS OF VISUAL PERCEPTION AND IMAGING APPLICATIONS
SORBONNE UNIVERSITÉ PARIS, NOV. 2018
\[ x_0 \]

\[ x_{\text{MIN}} \]

\[ x_{\text{MAX}} \]
GEOMETRY OF THE SPACE OF VISUAL STIMULI: PSYCHOPHYSICS AND NEURAL MODELS

1. Space is more than color!
2. Geometry may make you a star! SSIM
3. Geometry and neural models (I)
4. Geometry is more than deep-nets
5. Some psychophysics for you!
6. Geometry and neural models (II) DS DOWNLOAD!
7. Conclusions
1. Space is more than color!

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Space is more than color! Dimensionality & Distance
Space is more than color!

Environment A: CIE D65 illumination

Environment B: CIE A illumination

Physics → Manifolds
Dimensions = Sensors

http://isp.uv.es/data_color.htm

Laparra & Malo  Neural Comput. 2012
Laparra & Malo Front. Neurosci. 2015

DOWNLOAD!
Space is more than color!

Physics $\rightarrow$ Manifolds
Dimensions $\in$ Sensors

Malo & Gutierrez Network 2006

Local ICA
Space is more than color!

Adaptation = local basis

Stimulation of Gabor sensor in different environments

Environment 1: No background
Environment 2: Striped background

Sequential PCA

Space is more than color!

Adaptation = local basis

\[ r = \mathcal{S}(x) = C \cdot \int_{x^0}^{x} \nabla \mathcal{S}(x') \cdot dx' = C \cdot \int_{x^0}^{x} D(x') \cdot \nabla U(x') \cdot dx' \]

\[ r_i = C_{ii} \cdot \int_{x^i_{\perp}}^{x^i_{\perp}} D(x') \cdot \nabla U(x') \cdot dx' = C_{ii} \int_{0}^{u_{i\perp}} p_{ui}(u') \gamma \, du' \]

Laparra & Malo, Neural Comp. 2012
Laparra & Malo, Front. Neurosci. 2015
Sequential PCA
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- Dimensionality & Distance
- Physics/Manifold & Sensors
- Adaptation ≠ local basis
1. Space is more than color!
2. Geometry may make you a star! the SSIM index
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7. Conclusions
Geometry may make you a star! The image quality community.
2) Geometry may make you a star! Euclides vs SSIM

\[
\rho_p = 0.43 \quad \rho_s = 0.55 \quad \rho_k = 0.4
\]

- RMSE
- Euclidean Distance

\[ y = x + \Delta x \]

\[ d = \| \Delta x \|_2 \]
Geometry may make you a star! Euclides vs SSIM

Image Quality Assessment: From Error Visibility to Structural Similarity
Zhou Wang, Member, IEEE, Alan C. Bovik, Fellow, IEEE
Hamid R. Sheikh, Student Member, IEEE, and Eero P. Simoncelli, Senior Member, IEEE

Fig. 1. A prototypical quality assessment system based on error sensitivity. Note that the CSF feature can be implemented either as a separate stage (as shown) or within “Error Normalization”.

A. New Philosophy

Fig. 3. Diagram of the structural similarity (SSIM) measurement system.

RMSE
Euclidean Distance

\[ y = x + \Delta x \]

\[ d = \| \Delta x \|_2 \]
Euclidean vs SSIM

\[ \text{Euclid} = |x - y| \]

\[ \text{SSIM} \propto \frac{\sigma_y + k}{\sigma_x \sigma_y + k} \]

Fig. 4. Three example equal-distance contours for different quality metrics. (a) Minkowski error measurement systems; (b) component-weighted Minkowski error measurement systems; (c) magnitude-weighted Minkowski error measurement systems; (d) magnitude and component-weighted Minkowski error measurement systems; (e) the proposed system (a combination of Eqs. (9) and (10)) with more emphasis on \( s(x, y) \); (f) the proposed system (a combination of Eqs. (9) and (10)) with more emphasis on \( c(x, y) \). Each image is represented as a vector, whose entries are image components. Note: this is an illustration in 2-D space. In practice, the number of dimensions should be equal to the number of image components used for comparison (e.g., the number of pixels or transform coefficients).
**Euclides vs SSIM**

Geometry may make you a star!

- RMSE: Euclidean Distance

$$d = \| \Delta x \|_2$$

$$x + \Delta x$$

- SSIM

67th EMMY Engineering Award of the American Television Academy 2015

![YouTube Video](https://youtu.be/e5-LCFGdgMA)
Geometry may make you a star! Euclides vs SSIM
(2) Geometry may make you a star!

the review

What about proper neural models?
(such as Carandini & Heeger Divisive Normalization)

\[ x \xrightarrow{S} r \xleftarrow{S^{-1}} \]
Geometry may make you a star!

Proper neural models $S$ also give the right input dependent behavior!
1. Space is more than color!

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3. Geometry and neural models (I)

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6. Geometry and neural models (II)

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- Image Quality
- Euclides vs SSIM
- Don't forget S!
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(3) Geometry and neural models (I)

Euclide is right (in the proper domain)

Contrast:
- 0
- 0.1
- 0.2
- 0.3
- 0.4
- 0.5
- 0.6

Visibility:

\[ \Delta r \]

\[ \Delta C \]
3. Geometry and neural models (I)

Euclidean distance in the response domain + Distance preservation under transforms + Taylor

\[ d^2(r, r + \Delta r) = \Delta r^T \Delta r \]

\[ d^2(x, x + \Delta x) = \Delta x^T P S^T D S \Delta x \]

\[ \Delta r = D_x S(x) \Delta x \]

Non trivial metric! Jacobian of neural model!

Malo et al. Disp. 99, IEEE TIP 06, JOSA 10, PLOS 18
3. Geometry and neural models (I)

Divisive Normalization neural models — Dynamic models (e.g. Wilson-Cowan)

[MaLo & Bertalmio arXiv 18]

\[ S \]

\[ x \xrightarrow{L} e \xrightarrow{N} r \]

\[ e = T \cdot x \]

\[ r = k \cdot \frac{e}{b + H \cdot e} \]

\[ \nabla_x S \sim (I - D_r H) \cdot D_e \cdot T \Rightarrow g = \nabla_x S^\top \nabla_x S \]

\( T = \) wavelet basis matrix
\( e = \) wavelet vector
\( b = \) semisaturation
\( H = \) interaction kernel
\( k = \) constant - dyn. range

Masking and adaptation

Non diagonal! Input dependent!
Geometry and neural models (I)

Divisive Normalization
Martínez, Berdimuhamedov & Malo. PLoS 2018

COLOR
- Spectral integration
- Adaptation
- Opponency
- Saturation

SPATIAL TEXTURE
- Contrast
- CSFs
- Global masking
- Wavelet
- Cross-band masking

CASCADE $L + N$
3. Geometry and neural models (I)

$\rho_p = 0.43 \quad \rho_s = 0.55 \quad \rho_k = 0.4$

$\rho_p = 0.6 \quad \rho_s = 0.63 \quad \rho_k = 0.45$

SSIM

RMSE
Euclidean Distance

$\rho_p = 0.83 \quad \rho_s = 0.83 \quad \rho_k = 0.64$

$\rho_p = 0.81 \quad \rho_s = 0.78 \quad \rho_k = 0.59$

$\rho_p = 0.82 \quad \rho_s = 0.78 \quad \rho_k = 0.59$

$\rho_p = 0.88 \quad \rho_s = 0.88 \quad \rho_k = 0.69$

V1_model_DCT_DN_color  V1_model_wavelet_DN_color  BioMultiLayer_L_NL_color  BioMultiLayer_L_NL_color


IEEE Trans. Im. Proc. 2006 ....

Front. Neurosci. 2018 a

PLoS ONE 2018

Divisive Normalization

Martínez, Bertalmio & Malo PLoS 2018

Laparra & Simoncelli JOSA 2017

67th EMMY Engineering Award of the American Television Academy 2015 !

Paper 2004
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\[ g(x) = \nabla S(x)^T \nabla S(x) \]

Excellent behavior!
1. Space is more than color!
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3. Geometry and neural models (I)
4. **Geometry is more than deep-nets**
5. Some psychophysics for you!
6. Geometry and neural models (II)
7. Conclusions
Geometry is more than deep-nets

[Proper neural models is more than regression]

\[(T) \quad (H)\]

WAVELET - KERNEL BALANCE!

\[
r = k \cdot \frac{T \cdot x}{\left[ b + \left[ D_\mu \cdot H \cdot D_r \right] \cdot T \cdot x \right] \cdot H}
\]
Geometry is more than deep-nets

[Proper neural models is more than regression]

WAVELET - KERNEL BALANCE!


4.1 Expected behavior

4.2 Naive Divisive Normalization

Malo et al. Neural Comp. 2010
JCSA A 2010
PloS ONE 2018

4.3 Unit norm Watson & Solomon Kernel

Watson & Solomon JCSA A 1997

4.4 Gauges for by-hand tuning
4. Geometry is more than deep-nets
[Proper neural models is more than regression]

4.1 Expected behavior
Geometry is more than deep-nets

[Proper neural models are more than regression]

4.1 Expected behavior

LOW

HIGH

Cavanagh 00

\[ C_M = 0 \]

\[ C_M \ll \]

\[ C_M \gg \]
Geometry is more than deep-nets
[Proper neural models is more than regression]

4.2 Naive Divisive Normalization

\[ r = k \frac{e}{b + \left[ D_e \cdot H \right] \cdot e} \]
4. Geometry is more than deep-nets
   [Proper neural models is more than regression]

4.2 Naive Divisive Normalization

Malo et al. JGSA A 2010
PLoS 2018

Performance of MODEL – A (compared to SSIM)
4.2 Naive Divisive Normalization

Geometry is more than deep-nets
[Proper neural models is more than regression]

\[ y = 1 \equiv \text{NOTHING} \]
4. Geometry is more than deep-nets
[Proper neural models is more than regression]

4.3 Use unit-norm Gaussian Kernel  Watson & Solomon JOSA 97

Keep $\Gamma_x$ and
$\sigma_\theta \sim 30^\circ$
$\sigma_f \sim 1$ octave
4. Geometry is more than deep-nets
[Proper neural models is more than regression]

4.3 Use unit-norm Gaussian Kernel

Watson & Solomon JOSA 87
4. Geometry is more than deep-nets

[Proper neural models is more than regression]

4.4 By-hand tuning

\[ r = k \cdot \frac{\mathbf{T} \cdot \mathbf{x}}{b + \left[ \mathbf{D}_e \cdot \mathbf{H} \cdot \mathbf{D}_r \right] \cdot \mathbf{T} \cdot \mathbf{x}} \]

\[ H = \mathbf{D}_e \cdot \mathbf{H}_{\text{GAUSS}} \cdot \mathbf{D}_r \]
Geometry is more than deep-nets
[Proper neural models is more than regression]

4.4 By-hand tuning

OK!
4. Geometry is more than deep-nets

[Proper neural models is more than regression]

4.4 By-hand tuning

\[ H = D_e \cdot H_{\text{GAUSS}} \cdot D_r \]
Equivalence of Divisive Normaliz. & Wilson-Cowen

The question
Where does this come from?

\[ H = D_e \cdot H_{\text{GAUSS}} \cdot D_r \]
Equivalence of Divisive Normaliz. & Wilson-Cowan

Assumptions:

\[ \dot{x} = e - D_\alpha x - W \cdot f(x) \]

* Equivalence in steady state

* Piece-wise linear \( f(\cdot) \) in WC

\[ e = D_\alpha x + W \cdot f(x) \]

\[ \Rightarrow \quad e = (D_\alpha + W) \cdot x \]

* Truncation of inverse in DN

\[ (I - D_{(\frac{K}{K})}^x)(H)^{-1} = I + \sum_{n=1}^{\infty} (D_{(\frac{K}{K})}^x)(H)^n \approx I + D_{(\frac{K}{K})}^x(H) \]

\[ e = (1 - D_{(\frac{K}{K})}^x)(H)^{-1} D_{(\frac{K}{K})} x \quad \Rightarrow \quad e = (D_{(\frac{K}{K})} + D_{(\frac{K}{K})}^x(H)) \cdot x \]
Equivalence of Divisive Normaliz. & Wilson-Cowan

Parameters of Div. Norm. from Wilson-Cowan

\[ b = k \cdot \alpha \]

\[ H = D_{(k/x)} \cdot W \cdot D_{(k/b)} \]

AHA!

\[ H = D_e \cdot H_{\text{GAUSS}} \cdot D_r \]

- Signal dependence
- Wiring
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- No blind fitting!
- Balance problem
- Always check visibility
- Relation DN – WC
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Some psychophysics for you!

Download! Basic facts to falsify models

http://isp.uv.es/code/visioncolor/vistamodels

Frequency (CSFs)

Contrast (mask)

Noise
1. Space is more than color!
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6. Geometry and neural models (II) \[ g = \mathcal{A} \mathcal{S}^* \mathcal{A} \mathcal{S} \]
7. Conclusions
(6) Geometry and neural models (II)

More refined geometry-based psychophysics

Maximum differentiation

\(\nabla_x S\)

Falsify models (or parameters)

By assessing model-based extreme distortions
Geometry and neural models (II)

More refined geometry-based psychophysics

Maximum differentiation

$\nabla_x S$
Geometry and neural models (II)

More refined geometry-based psychophysics

Maximum differentiation

The original algorithm
[Wang & Simancelli: JoV 2008]

The approximation
[Malo & Simancelli: SPIE 2013
Martina et al. PloS 2018]

\[ g(x) = \nabla_x S(x)^T \cdot \nabla_x S(x) \]
Geometry and neural models (II)

Iterative: Same energy, progressively different visibility

Maximum different
Also download D x S!
Geometry and neural models (II)
Geometry and neural models (II)

Iterative

Eigen vectors of metric

MIN VISIB.

MAX VISIB.

MIN VISIB.

MAX VISIB.
Geometry and neural models (II)

iterative

Eigen vectors of metric
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Conclusions

Derivatives and inverse of Linear+Nonlinear Neural models
http://arxiv.org/abs/1711.00526

* Neural models are relevant for geometry
  \[ g(x) = \nabla_x S(x)^T \cdot \nabla_x S(x) \]
* Metric ⇒ New psychophysics
  Maximum Differentiation
* Other implications of \( \nabla_x S \)
  - Adaptive receptive fields
  - Information theory
  - Decoding

In praise of artifice reloaded  http://arxiv.org/abs/1801.09632

* Do not trust blind optimization
  - Check with psychophysics
  - Architecture changes may be required


* Divisive Normalization from Wilson-Cowan